Short Paper: On Stabilizing the Staking Rate, Dynamically Distributed Inflation and Delay Induced Oscillations

Carlo Brunetta[1], Amit Chaudhary[2],
Stefano Galatolo[3], and Massimiliano Sala[4]

[1] Independent Researcher, France, brunocarletta@gmail.com
[2] University of Warwick, UK, amit.chaudhary.3@warwick.ac.uk
[3] University of Pisa, Italy stefano.galatolo@unipi.it
[4] University of Trento, Italy massimiliano.sala@unitn.it

Abstract. Dynamically distributed inflation is a common mechanism used to guide a blockchain's staking rate towards a desired equilibrium between network security and token liquidity. However, the high sensitivity of the annual percentage yield to changes in the staking rate, coupled with the inherent feedback delays in staker responses, can induce undesirable oscillations around this equilibrium.

This paper investigates this instability phenomenon. We analyze the dynamics of inflation-based reward systems and propose a novel distribution model designed to stabilize the staking rate. Our solution effectively dampens oscillations, stabilizing the yield within a target staking range.

Keywords: Tokenomics · Dynamically Distributed Inflation · Delay Induced Oscillation

1 Introduction

A fundamental economic design challenge for Proof-of-Stake (PoS) [3, 18] protocols is maintaining the staking rate within a specific optimal range [11, 13, 14]. To reach this target, it is essential to find a trade-off between network security and token liquidity. A staking rate below the desired threshold reduces the crypto-economic security of the consensus mechanism by lowering the cost of a malicious attack. Conversely, an excessively high staking rate impairs the protocol's economic vitality by diminishing the circulating supply, which can lead to low on-chain liquidity, increased price volatility, and a constrained transactional ecosystem. Therefore, designing incentive mechanisms to guide the staking rate into some desired target corridor is a primary objective of a protocol's monetary policy. To achieve this balance, many modern blockchains implement a dynamic inflation mechanism where rewards distributed to stakers are a function of the current staking rate, maximizing the individual yield as this rate approaches the desired value [6, 17]. This approach of adjusting rewards based on the current state of the network is conceptually analogous to the “flexible inflation targeting” rules studied in modern monetary economics, where central banks manage a trade-off between inflation stability and output-gap stabilization [10].

As an example of dynamical-inflation protocols in the PoS context, Polkadot have implemented reward mechanisms that tie stakers’ rewards to token inflation, following a non-linear curve designed to maximize the inflation distributed to stakers at the target *staking rate* [2, 7] (see Fig. 1). While an approach of this kind is effective in creating a strong incentive towards an equilibrium point, its architecture presents *dynamical vulnerabilities.* Indeed, the yield curve typically exhibits a very steep gradient (high sensitivity) around the target point meaning that a small deviation in the *staking rate* causes a sharp change in the yield, stimulating a strong corrective action in the economic agents. When coupled with high sensitivity, the staker reaction to these yield changes becomes a natural *delayed feedback* that, as it will be discussed in this work, can create the conditions for the emergence of oscillations in the staking rate.

Economic literature has identified how the combination of high reactivity and decisional lags can generate cycles and instability in dynamical systems [9, 12]. In the context of staking, this delay is both structural and behavioural. Recent literature indeed highlights that the design of staking mechanisms involves inherent delays, such as lock-up periods to ensure validator quality [16] and managed exit queues to preserve protocol security [15]. Further empirical evidence confirms this dynamics, showing that the reward rate in one period is a significant predictor of an increase in the *staking rate* in the next period. Moreover, empirical analysis indicates that the combined structural and behavioral delay in staker responses is of the order of one week [8]. As we will see in the following, this delayed reaction can cause the system to continuously over-correct, oscillating around the desired equilibrium rather than converging to it stably.

In this work, however we focus on the study of the effect of the sensitivity and the structural delay, showing that this is sufficient to generate oscillations in the staking rate. We study this phenomenon by modeling the aggregate behavior of economic agents as a single "mean" staker, who adjusts their staking position in proportion to the difference between the current yield and a target yield level. While this target yield is held constant in our model, we acknowledge it is influenced by external factors, such as alternative market investments. In Sec. 3, through numerical simulations, we compare the evolution of the staking rate across three distinct inflation distribution schemes, showing how oscillations can be dampened by the design of an appropriate inflation curve. In addition to these simulations, by a suitable linearization of the system around its equilibrium points, in Sec. 4 we present analytical results that quantitatively connect the emergence of oscillations to the derivative of the inflation curve at the optimal staking point.

Given that the delays are a necessary structural feature and, as we have seen, a primary driver of the oscillatory dynamics, the above analysis is strongly motivated by the need to manage instability by focusing on the remaining control variable: the design of the dynamic inflation curve.

To assess the impact of the reward curve's design on system stability, we compare three inflation-distribution schemes. As benchmarks, we use a constant inflation model where rewards are independent of the staking rate and a Polkadot-style model characterized by a high-sensitivity yield peak. Against these, we test a particular inflation curve, which is engineered to reduce oscillations by replacing the yield peak with a robust "stability corridor". The core objective of this design is to nullify the yield's sensitivity within a predefined target range, thus removing the main driver of instability, while preserving strong corrective incentives outside this corridor.

Both numerical and analytical results confirm that the combined effect of the sensitivity of the stakers, delay and high derivative of the yield at the equilibrium points generate persistent oscillations, which can be dumped by reducing the derivative of the inflation at the equilibrium point.

Our approach of analyzing staking rate stability through the lens of control theory aligns with recent research applying formal methods and dynamical systems to blockchain economies. This literature has explored the impact of various factors on protocol stability, such as the competitive pressure from on-chain lending yields [5], the complex emergent behaviors in lending markets [4], or the adaptive management of token supplies for infrastructure networks [1].

2 Oscillations and Dynamical Distribution of the Inflation

In this section, we develop a dynamic model to quantitatively analyze the impact of the reward curve's design on the stability of the staking rate. The model simulates the aggregate behavior of stakers as a discrete-time process, where the change in staking is driven by the difference between the current yield and a target yield level Y_{t}, incorporating a reaction delay consistent with empirical observations. We utilize this framework to compare the system's evolution under three distinct inflation distribution schemes: a baseline constant-inflation model, a Polkadot-inspired model featuring a high-sensitivity yield peak, and a third mechanism based on a stability corridor designed to stabilize the convergence. Through numerical simulations, we will observe the time trajectory of the staking rate for each scenario, and the related presence, amplitude, of damping of oscillations, thereby assessing the effectiveness of each design in promoting a stable equilibrium.

We now describe the three dynamical inflation distribution mechanisms we compare. We will denote by σ the current staking rate, i.e. the fraction of the total supply being staked and by $I(\sigma)$ the distributed total inflation, depending on σ. Fig. 1 graphically illustrates the three distributed-inflation functions and the relative Annual Percentage Yield (APY).

Constant Reward. In this reward scheme, $I(\sigma)$ is constant. We set it at 0.1 (10%) in our study, and this will be the maximum inflation rate also provided by the other dynamic-reward mechanisms studied here. Keeping constant the total distributed inflation, the individual yield per staker, $Y(\sigma)$, is inversely

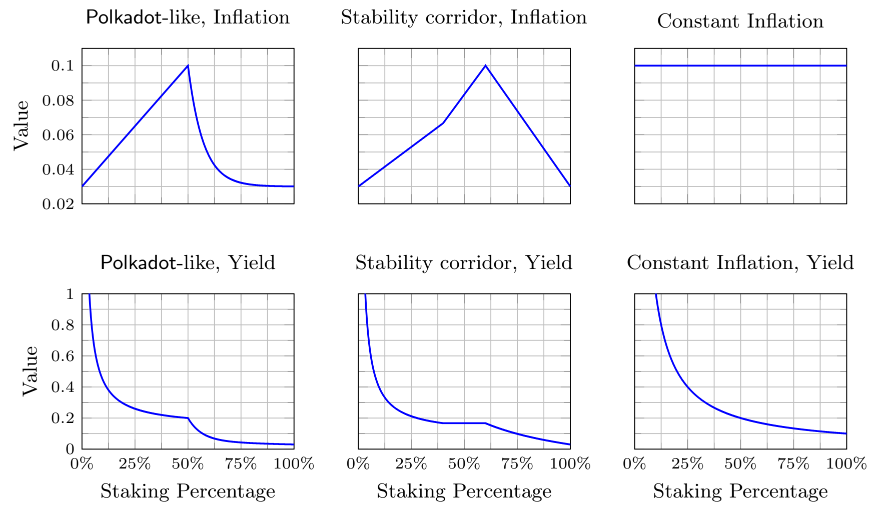

Fig. 1: The total inflation curve $I(\sigma)$ as a function of the staking rate σ for the three examples described in the text.

proportional to the staking rate, $\sigma \in [0, 1]$, as the same amount of rewards is divided among a larger staking base according to $Y(\sigma) = \frac{I_{\mathsf{tot}}(\sigma)}{\sigma}$.

In a simple situation like this one, the equilibrium staking rate σ_{eq} (where the individual yield equals the target yield Y_{t}) is then $\sigma_{\mathsf{eq}} = \frac{I_{\mathsf{tot}}}{Y_{\mathsf{t}}}$. There can be oscillations around this fixed point, depending on the sensitivity of the behavior of the stakers, as it will be discussed in the next sections.

Polkadot-like Reward. A well-known example of a dynamic-reward mechanism is the Polkadot inflation model, which utilizes a piecewise function to steer the staking rate towards an ideal equilibrium. The model's behaviour is governed by several key parameters:

- σ_{id}: the target staking rate (in our study it will be set to 0.5).
- I_0: the minimum annual inflation rate (which we set to 0.03).
- I_{M}: the maximum inflation rate, reached when $\sigma = \sigma_{\mathsf{id}}$ (we set to 0.10).
- d: a decay parameter controlling how quickly inflation decreases for staking rates above the ideal target (which we set to 0.05).

The total inflation function is formally defined by the following equation:

$$I(\sigma) = \begin{cases} I_0 + \left(\frac{\sigma}{\sigma_{\mathsf{id}}}\right) \cdot (I_{\mathsf{M}} - I_0) & \text{if } 0 < \sigma \leq \sigma_{\mathsf{id}} \\ I_0 + (I_{\mathsf{M}} - I_0) \cdot 2^{\left(\frac{\sigma_{\mathsf{id}} - \sigma}{\mathsf{d}}\right)} & \text{if } \sigma > \sigma_{\mathsf{id}}. \end{cases} \tag{1}$$

The function consists of two distinct regimes. For staking rates at or below the target, inflation increases linearly, creating a progressively stronger incentive to stake. Conversely, for staking rates above the ideal target, inflation decays exponentially, which strongly disincentivizes further staking to protect the token's liquidity and utility within the ecosystem. However as we will see, this rapid decrease of the reward, coupled with the delayed action of the stakers easily stimulate oscillations in the staking rate, we will discuss in next section.

Stability Corridor Reward. This inflation distribution system is based on a piecewise-affine total inflation function, defined by three distinct regimes.

We assume a target range for the *staking rate* between 40% and 60%. The inflation function will be constructed in a way that in this range the yield is constant. As before, we set the inflation to range between a minimum of 3% to a maximum of 10% and to vary continuously with three affine branches. These three affine branches will be constituted by an **incentivization phase** for $0 \leq \sigma < 0.4$, where the inflation grows linearly and the associated yield per token is very high promoting the investment. Then, for $0.4 \leq \sigma \leq 0.6$, we have a **stability corridor** in this interval, inflation continues to grow linearly, but in a way that the individual yield is constant. This is the core of our stabilization mechanism. By nullifying the change in yield, the sensitivity that, combined with delay, generates instability is eliminated. Agents have no incentive to alter their positions in response to fluctuations of the *staking rate* within this zone, thus promoting a stable equilibrium. Then, we have for $0.6 < \sigma \leq 1.0$ a **disincentivization phase** where to prevent excessive token illiquidity, the inflation decreases linearly from its 10% peak back to the base value $I_{\mathsf{b}} = 3\%$ when $\sigma = 1.0$ and the yield also decreases similarly, discouraging further staking.

To satisfy the above requirements, $I(\sigma)$ is defined as follows:

$$I(\sigma) = \begin{cases} 0.03 + 0.025 \cdot \sigma & \text{if } 0 \leq \sigma < 0.4 \\ 0.1 \cdot \sigma & \text{if } 0.4 \leq \sigma \leq 0.6 \\ 0.105 - 0.075 \cdot \sigma & \text{if } 0.6 < \sigma \leq 1.0 \end{cases} \tag{2}$$

3 A Dynamical Model for the Staking Rate Evolution

The preceding discussion suggests that the sensitivity of the yield to variations in the staking rate, combined with the delayed response of economic agents, may promote oscillatory behaviour around the equilibrium staking rate. In this section, we develop a model to illustrate, at least qualitatively, the emergence of this phenomenon and to assess the plausibility that this dynamic is an important driver of instability.

We simulate the aggregate behaviour of stakers under each of the three dynamic reward mechanisms described previously. We model the "mean" agent as deciding to increase or decrease their staked position in proportion to the difference between the current yield and a constant target yield. This decision is then implemented with a seven-day delay, a lag consistent with empirical observations [7]. The model's dynamics are simulated and analyzed for the three inflation distribution curves outlined in the previous section, using a target yield parameter of 16.6%.

We formalize the above ideas, measuring the time in days. The staking rate at time n will be denoted by $\sigma_n \in [0,1]$. The value of σ_n is determined by the value on the previous day, σ_{n-1}, plus an adjustment term which is proportional to the difference between the yield observed seven days prior, $Y(\sigma_{n-7})$, and a constant target yield, Y_t. The proportionality constant, $b > 0$, represents the sensitivity of the "mean" staker to yield deviations. We hence get the following recurrence relation, where the result is clamped to the valid range $[0,1]$:

$$\sigma_n = \max\Big(0, \min\big(1, \sigma_{n-1} + \sigma_{n-7}\left(Y\left(\sigma_{n-7}\right) - Y_\mathsf{t}\right) \cdot b\big)\Big) \tag{3}$$

It is important to note that, since the yield function Y is non-linear, the model described in Eq. (3) is a *non-linear dynamical system with delay*[5]. The model's global dynamics can hence be complex and difficult to study rigorously.

Our first method of analysis will therefore be numerical simulation. Fig. 2 illustrates the dynamics of the staking rate resulting from the simulation of our model under the three inflation-distribution schemes previously introduced: the Polkadot-like model, the stability corridor model and the constant inflation model. All simulations share similar base parameters: they start from an initial condition of $\sigma_0 = 1\%$, while the sensitivity parameter is set to $b = 0.6$ and the target yield is $Y_\mathsf{t} = 16.6\%$. The results show that the Polkadot-like model produces strong and persistent oscillations around the optimal rate. In contrast, the stability corridor model effectively dampens this oscillatory behavior, converging smoothly to a stable equilibrium within the target range. The constant inflation model exhibits an intermediate behavior, showing some initial oscillations that are eventually damped as the system settles into its final state.

4 Stability and Oscillations near the Equilibrium Points

To analytically investigate the emergence of oscillations, we study the local stability of the model's equilibrium points. The model's adjustment term can be rewritten, revealing an alternative economic interpretation.

Denoting by $\Delta\sigma_n \coloneqq \sigma_n - \sigma_{n-1}$ the change in the staking, we get:

$$\Delta\sigma_n \;=\; b \cdot \big(Y\left(\sigma_{n-7}\right)) - Y_\mathsf{t}\big) \cdot \sigma_{n-7} \;=\; b \cdot \big(I\left(\sigma_{n-7}\right) - Y_\mathsf{t} \cdot \sigma_{n-7}\big) \;. \tag{4}$$

[5] An alternative, simpler model of the same kind could be considered where the adjustment term is not scaled by the staking rate. The dynamic would then be described by the equation $\sigma_n = \sigma_{n-1} + b \cdot (Y(\sigma_{n-7})) - Y_\mathsf{t})$. This represents a system where the absolute change in the staking rate is proportional to the absolute yield gap, implying that the pool of capital ready to react to yield deviations is constant, regardless of the current number of stakers. Although more direct, we believe the model presented in the main text is more realistic, particularly at low staking rates, where new investors may be reluctant to invest in the system.

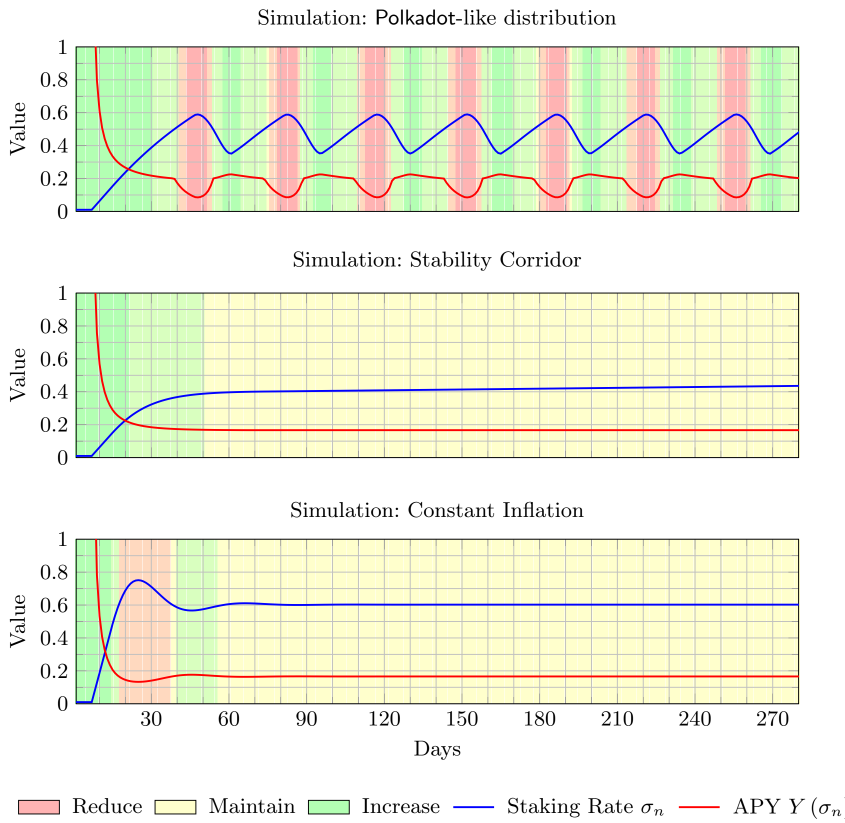

Fig. 2: The dynamics of the staking rate with the three proposed inflation distribution curves. In blue we plot the staking rate as a function of time, in red the relative yield. The actions taken by the staker (increase, maintain or reduce staking) are plotted as background colour.

The equilibrium points of the system, denoted by σ^*, are steady states, where the staking rate no longer changes. They are found by setting $\Delta\sigma_n = 0$, which yields the equilibrium condition $Y(\sigma^*) = Y_{\mathsf{t}}$. To analyze the stability around such an equilibrium, we linearize the system. Let $\sigma_n = \sigma^* + x_n$, where x_n is a small perturbation. By applying a first-order Taylor expansion to the model, we obtain a linear difference equation that governs the evolution of the perturbation:

$$x_n = x_{n-1} + b \cdot (I'(\sigma^*) - Y_{\mathsf{t}}) \cdot x_{n-7} \tag{5}$$

where $I'(\sigma^*)$ is the derivative of the inflation function evaluated at the equilibrium point. The corresponding characteristic equation is derived by assuming that the solution to the linear difference equation, $x_n = x_{n-1} + K_{\mathsf{eq}} \cdot x_{n-7}$, is of the exponential form $x_n = \lambda^n$. This is a seventh-degree polynomial in λ, i.e. $\lambda^7 - \lambda^6 - K_{\mathsf{eq}} = 0$, where we have defined the stability parameter K_{eq} as

$K_{\mathsf{eq}} = b(I'(\sigma^*) - Y_{\mathsf{t}})$. The equilibrium is locally stable if and only if all roots of this equation lie inside the unit circle ($|\lambda| < 1$). The stability boundary is reached when a root lies on the unit circle. Analytical investigation reveals that the system is stable for K_{eq} within the range:

$$-2\sin\left(\frac{\pi}{26}\right) < K_{eq} < 0 \tag{6}$$

which provides the approximate numerical stability condition $-0.241 < K_{eq} < 0$. This criterion quantitatively links the stability of the system to the properties of the inflation curve at its equilibrium points, particularly its slope, $I'(\sigma^*)$.

We now apply the analytical stability criterion, $-0.241 < K_{eq} < 0$, to the three inflation models, using a target yield of $Y_{\mathsf{t}} = 16.6\%$. For the constant inflation model, the equilibrium occurs at $\sigma^* \approx 60.2\%$. Since the derivative $I'(\sigma^*)$ is zero, the stability parameter is $K_{\mathsf{eq}} = -0.166 \cdot b$, and the system remains stable for a wide range of staker sensitivity ($0 < b < 1.452$). In contrast, for the Polkadot-style model, the equilibrium point at $\sigma^* \approx 51.6\%$ falls on the steep, exponentially decaying part of the inflation curve. The large negative derivative, $I'(\sigma^*) \approx -0.772$, yields a stability parameter of $K_{\mathsf{eq}} = -0.938 \cdot b$. Consequently, the system is stable only for a very narrow sensitivity range ($0 < b < 0.257$), making it highly prone to oscillations. Finally, for our proposed stability corridor model, the equilibrium at $\sigma^* \approx 21.3\%$ is located on the first, gently sloped branch. The resulting small positive derivative, $I'(\sigma^*) = 0.025$, leads to $K_{\mathsf{eq}} = -0.141 \cdot b$ and a very wide stability range ($0 < b < 1.709$), confirming that this design is an even more robust against oscillatory behaviours.

5 Conclusions and Future Directions

In this paper, we have investigated the dynamic instability of staking rates in Proof-of-Stake protocols. We demonstrated, through a simple discrete-time model, studied by simulation and analytical investigation that the interplay between yield sensitivity and delayed staker responses is a powerful mechanism that can, by itself, generate significant and persistent oscillations around the target equilibrium.

It is important to emphasize that this work is intended as a proof of concept. The simulations presented are meant to qualitatively illustrate the stabilization effect of our proposed dynamical-inflation function. While the seven-day delay reflects the average lag observed by Cong *et al.* [8] (and in any case it can vary from system to system) other model parameters, such as the staker sensitivity b, have been chosen to reflect plausible agent behavior but are not yet supported by systematic empirical studies. Further research is necessary to refine these behavioural models and to statistically estimate their parameters from on-chain data of specific decentralized economies.

Acknowledgments This paper has been written during the first research phase for the Palliora project (`https://www.palliora.org`), which incorporates our staking approach for the token inflation.

References

1. Akcin, O., Streit, R.P., Oommen, B., Vishwanath, S., Chinchali, S.: A Control Theoretic Approach to Infrastructure-Centric Blockchain Tokenomics, (2022). https://doi.org/10.48550/ARXIV.2210.12881.
2. Alfonso Cevallos, J.G.: Token Economics, (2023). https://research.web3.foundation/Polkadot/overview/token-economics (visited on 08/12/2025).
3. Bano, S., Sonnino, A., Al-Bassam, M., Azouvi, S., McCorry, P., Meiklejohn, S., Danezis, G.: SoK: Consensus in the Age of Blockchains. In: Proceedings of the 1st ACM Conference on Advances in Financial Technologies. AFT '19, pp. 183–198. ACM (2019). https://doi.org/10.1145/3318041.3355458
4. Bartoletti, M., Lipparini, E.: A theory of Lending Protocols in DeFi, (2025). https://doi.org/10.48550/ARXIV.2506.15295.
5. Chitra, T.: Competitive equilibria between staking and on-chain lending, (2020). https://doi.org/10.48550/ARXIV.2001.00919.
6. Clayton, C., Schaab, A.: A Theory of Dynamic Inflation Targets. American Economic Review **115**(2), 448–490 (2025). https://doi.org/10.1257/aer.20230496
7. Cong, L., He, Z., Tang, K.: Staking, Token Pricing, and Crypto Carry. SSRN Electronic Journal (2022). https://doi.org/10.2139/ssrn.4059460
8. Cong, L.W., He, Z., Tang, K.: The Tokenomics of Staking (2025)
9. Ezekiel, M.: The Cobweb Theorem. The Quarterly Journal of Economics **52**(2), 255 (1938). https://doi.org/10.2307/1881734
10. Giannoni, M., Woodford, M.: "Optimal Inflation-Targeting Rules". In: The Inflation-Targeting Debate. University of Chicago Press, Dec. 2004. http://www.nber.org/chapters/c9557.
11. John, K., Rivera, T., Saleh, F.: Equilibrium Staking Levels in a Proof-of-Stake Blockchain. SSRN Electronic Journal (2021). https://doi.org/10.2139/ssrn.3965599
12. Kaldor, N.: A Classificatory Note on the Determinateness of Equilibrium. The Review of Economic Studies **1**(2), 122 (1934). https://doi.org/10.2307/2967618
13. Kogan, L., Fanti, G., Viswanath, P.: Economics of Proof-of-Stake Payment Systems. SSRN Electronic Journal (2023). https://doi.org/10.2139/ssrn.4320274
14. Leporati, A., Rovida, L.: Looking for stability in proof-of-stake based consensus mechanisms. Blockchain: Research and Applications **5**(4), 100222 (2024). https://doi.org/10.1016/j.bcra.2024.100222
15. Neuder, M., Pai, M., Resnick, M.: Optimizing Exit Queues for Proof-of-Stake Blockchains: A Mechanism Design Approach, (2024). https://doi.org/10.48550/ARXIV.2406.05124.
16. Oderbolz, N., Marosvölgyi, B., Hafner, M.: Towards an Optimal Staking Design: Balancing Security, User Growth, and Token Appreciation. (2024). https://doi.org/10.48550/ARXIV.2405.14617
17. Riposo, J., Gupta, M.: A Crypto Yield Model for Staking Return. FinTech **3**(1), 116–134 (2024). https://doi.org/10.3390/fintech3010008
18. Saleh, F.: Blockchain without Waste: Proof-of-Stake. The Review of Financial Studies **34**(3), 1156–1190 (2020). https://doi.org/10.1093/rfs/hhaa075